Solving Differential Equation by Modified Genetic Algorithms

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Abstract

Differential equation is a mathematical equation which contains the derivatives of a variable, such as the equation which represent physical quantities, In this paper we introduced modified on the method which propose a polynomial to solve the ordinary differential equation (ODEs) of second order and by using the evolutionary algorithm to find the coefficients of the propose a polynomial [1] .Our method propose a polynomial to solve the ordinary differential equations (ODEs) of nth order and partial differential equations(PDEs) of order two by using the Genetic algorithm to find the coefficients of the propose a polynomial ,since Evolution Strategies (ESs) use a string representation of solution to some problem and attempt to evolve a good solution through a series of fitness –based evolutionary steps .unlike (GA) ,an ES will typically not use a population of solution but instead will make a sequence of mutations of an individual solution ,using fitness as a guide[2] .Numerical example with good result show the accuracy of the our method compared with some existing methods .and the best error of method it's not much larger than the error in best of the numerical method solutions.

1- Introduction

Genetic Algorithms (or simply GAs) are great and usually applicable stochastic search and optimization ways based on the models of natural selection and natural evaluation. GAs work on a population of individuals represents candidate solutions to the optimization problem.[3] that creates better than random results [4]. This notes was first mathematically by John Holland in 1975 in his paper, "Adaptation in Natural and Artificial Systems" [5]. Drawing on Darwin's theory of evolution, which states "survival for the stronger,"Goldberg developed the genetic algorithm [6]. Darwin also stated that the survival of the organism can be maintained through the process of selection, crossover and mutation. Darwin's concept of evolution was modified in a mathematical algorithm to solve a problem called the objective function of the natural way [7]. Most of the different GA techniques emphasis on some difference in the standard genetic operations already present to improve the shortcomings of the standard GA. Change of the selection operation is a popular research subject trying to increase the GAs study of the solution search space and to avoid grouping or early convergence [8]. The basic concept of GAs is designed to simulate processes in natural system necessary for evolution [5]. GAs have been widely studied, experimented and applied in many fields of sciences [9]. Differential Equations (DEs) are an important concept in many branches of science [1]. In many states, where a differential equation is given with boundary conditions, an approximate solution is often obtained by using numerical methods [10]. By GA to find the solution for simple mathematical equality Problem this solution is called a chromosome [11]. Measured the suitability of solution which generated by GA by undergo this chromosomes to a process called fitness function [12]. Genetic algorithms generations solutions for solving Poisson Equation [13], new method for solving unstable differential equation [14], order to solve optimization problems such a technique may be used in order to solve differential equations [1], used to find roots of algebraic Equations [15]. accelerated (GAs) method to solve ordinary differential equations and use to find an explicit approximation [16], use as a nonlinear technique in solving linear and nonlinear equation system[17], used to find the solution of Systems of Simultaneous Non-Linear Equations [18], genetic algorithm technique introduce numerical algorithm, for solving a class of nonlinear systems of second-order boundary value problems [19], This study investigates the ability to apply the genetic algorithm to find solutions for DEs involving a new mathematical model used to find ODEs and PDEs. We also find the efficiency of our method when comparing the results obtained by using our method with numerical methods.

2- Genetic Operators

The term "Genetic operators" is applied to methods used to simulate nature in computer-based evolutionary systems .The genetic operators consist of selection, crossover and mutation [3]. Now to describe of each term:

2-1: Selection [20]

The method of selection plays an important part in population evolution, as it is a key aspect of the genetic algorithm for identifying individuals and then subjecting them to two processes crossover or mutated depending on their fitness. The selection should be completed to discover the characteristics of individuals with a high level of fitness while searching the total solution.

2-2: Crossover [8]

Crossover is the most important operator in GA and is responsible for production of new solutions. It is mostly done by taking two parent solutions and swapping two randomly selected parents' to produce from them a child. After the selection process, the population is enriched with better individuals.in this way, the newly created individual replaces the individual with the low suitability value, and the weak individual in the population are eliminated.

2-3: Mutation [9]

After crossover, the strings are subjected to mutation. The mutation plays the role of regaining lost genetic material and randomizing the genetic information Mutation is to randomly change one or more “genes” in a string that forms an individual’s chromosome according to their mutation probability. Mutation is performed to one individual to produce a new version of it, thus speeding up the process. In this method, one can face with the problem of elimination of good individuals. To prevent this, a structure is formed that ensures the best individuals are to be transmitted to the next generation.

3- Initial steps of genetic algorithms (Gas) [21]

The typical steps involved to design Genetic Algorithm, to be followed are:

Step1: randomly initialize population of size N and determine mutation rate p_m and crossover rate p_c

Step 2: determine fitness function of population N, which is used to measure the quality of an individual chromosome in the problem space .Fitness=$\frac{1}{1+fit(\vec{x})}$ where

$$fit(\vec{x}) = min \sum_{i=0}^{m}(G(x_i))^{2}$$

Step 3: compute the fitness values for each chromosome.

Step 4: Selection of pair of chromosomes from the select population for mating. By fitness with a crossover probability pc are selected the Parent chromosomes. Exchange parts of the two selected chromosome and create two children

Step 5:with the probability mutation p_m randomly change fitness values in the two children chromosome

Step 6: the resulting chromosome considers the new population

Step 7: Repeating step 4, step 5, and step 6 to be the new size of chromosomes population equal to N (initial population size).

Step 8: Replacement of the original parent chromosome population with the new produce population

Step 9: Repeating Step 4 through Step 8 until the termination criterion is satisfied.

4: Mathematical model of Ordinary Differential Equation

The ordinary differential equations (ODEs) are a differential equation containing derivatives of one or more dependent variables with respect to a single independent variable. In this section, we show the steps involved the mathematical model to solve ordinary differential equation (ODE) with initial condition

$$D\,(y^{(n)}, y^{(n-1)}, y^{(n-2)}, \ldots, y, t) = 0 \qquad (4.1) \qquad t \in [t_0, b]$$

$$y^{(k)}(t_0) = y_k \qquad k=0, 1, 2, \ldots, n-1$$

Where, n is degree of the differential equation and the initial condition is (n-1)

Let $x = \frac{t-t_0}{b-t_0}$ $\forall\ t \in [t_0, b]$

Then ODE (4 .1) with initial condition become

$D(y^{(n)}, y^{(n-1)}, y^{(n-2)}, . . . , y, x) = 0$ (4.2) $x \in [0,1]$

$y^{(k)}(0) = y_k$ k=0,1 ,2 , . .,n-1

Suppose the polynomial p(x) is a solution of equation (4 .2) where

$P(x) = \sum_{k=0}^{m} d_k\, \psi_k(x)$ (4. 3)

In special case $\psi_k(x) = x^k$ where d_k are coefficients of the polynomial and (m) is the degree of the polynomial Then

$P(x) = \sum_{k=0}^{m} d_k\, x^k$ (4.4)

$p^{(r)}(x) = \sum_{k=r}^{m} k(k-1) \dots (k-r+1)\, d_k\, x^{k-r}$ (4. 5)

$p^{(r)}(0) = y_k$ r=0, 1 , 2, …, m

Put the equation (4.4) and (4. 5) in the equation (4. 2) we get

G(x) =D $(p(x)^{(n)}, p(x)^{(n-1)}, p(x)^{(n-2)}, . . . , p(x), x) = 0$ (4.6)

The special case if ordinary differential equation (ODE) is linear differential equation (LDE) of order n^{th} and D $(y^{(n)}, y^{(n-1)}, y^{(n-2)}, . . . , y, x) = (\sum_{i=0}^{n} a_i(x) y^{(i)}(x)\ ) - f(x) = 0$ (4.7)

Put the equation (4.4) in (4.6) we get

$(G(x)\ = \sum_{i=0}^{n} a_i(x) \sum_{k=r}^{m} k(k-1) \dots (k-r+1) d_k x^{k-r}) - f(x) = 0$ (4.8)

Let $x_i = ih$ i=0, 1, 2… m where $h = \frac{1}{m}$, $x_i \epsilon [0, 1]$

to solve the ODE (4.8) to find the coefficient d_k k=0,1 ,…,m

represent each of the chromosome in a binary fixed point format That is, every coefficients is represented in binary assigning 1 bit to the sign, p bits to the integer part and q bits to the decimal part then the length of chromosome = (1+p+q) * (m-n+1)

Since t ∈ [0 ,1] if the length of the decimal part is δ for each d_k k= 0, 1 ,…, m

then the decimal part in bit is $q = \lceil \log_2 10^{\delta} \rceil$ (4.9)

and if the length of the integer part is σ for each d_k

k= 0, 1… m then the integer part in bit is $p = \lceil \log_2 10^{\sigma} \rceil$ (4. 10)

now compute the fitness function by using the form

Fitness = $\frac{1}{1+fit(\vec{x})}$ (4.11)

Where $fit(\vec{x}) = min \sum_{i=0}^{m} (G(x_i))^{2}$ (4. 12) $\vec{x} = (x_0 , x_1 , \dots, x_m)$

4-1: Example

$\bar{\bar{y}} - 100y = 0$ (4 .13)

$y(0)=0$, $\bar{y}(0) = 10$, $x \in [0 ,1]$

Since the analytic solution is $y = \frac{1}{2}(e^{10x} - e^{-10x})$

By our method let p(x) is a solution of the equation (4.13) and m=3 and n =2 by equation (4 .4) then $p(x) = \sum_{k=0}^{3} d_k x^k$

$p(x) = d_0 + d_1 x + d_2 x^2 + d_3 x^3$ from the initial condition we get $d_0=0$ and $d_1=10$

then $p(x) = 10x + d_2 x^2 + d_3 x^3$

since L= m+n-1 then L=2 also $h = \frac{1}{m}$ then h=0.333

$x_0=0$, $x_1=0.333$, $x_2=0.666$, $x_3=1$

$G(x_1)=\bar{\bar{p}}(x_1)- 100\, p(x_1)$

$G(x_2)=\bar{\bar{p}}(x_2)- 100\, p(x_2)$

$G(x)=(G(x_1))^2+(G(x_2))^2$

$G(x)=((2d_2+6d_3x_1)-100(10x_1+d_2x_1^2+d_3x_1^3))^2+((2d_2+6d_3x_2)-100(10x_2+d_2x_2^2+d_3x_2^3))^2=$

$((-9.108)\, d_2+2.2974d_3-333.333)^2+((-42.44)\, d_2-23.68\, d_3-666.666)^2$ by equation (4.12) then

$fit(\vec{x}) = \min(G(x))$ and by equation (4.11)

Fitness = $\frac{1}{1+fit(\vec{x})}$

to found the length of the chromosome by equations (4. 9) and (4.10) the binary assigning 1 bit , 6 bits to the integer part and the decimal part is 14 bits then the length of the chromosome is 28+1+6=35 random d_2 ,d_3 the format of d_2 ,d_3 is S6.28 with constraint $G(x_1)\geq 0$, $G(x_2)\geq 0$

Table (4-1) compare the error by our method with Runge – kutta method in example (4-1)

xi	The Absolute Error (Runge – kutta)	The Absolute Error by our method
0	0	0
0.1	0.009	4.43e-08
0.2	0.029	3.86e-09
0.3	0.112	4.99e-08
0.4	0.318	5.17e-08
0.5	1.348	5.81e-08
0.6	4.390	5.26e-08
0.7	13.894	5.81e-08
0.8	493.084	5.17e-08
0.9	131.516	5.29e-08
1	39.493	5.62e-08

4-2: example

$$\bar{\bar{y}} - 2\,\bar{y} + 2y = e^{2x} sinx \qquad (4.14)$$

$y(0)=-0.4$, $\bar{y}(0) = -0.6$, $x \in [0,1]$

Since the analytic solution is $y = -\frac{2}{5}e^{2x}cosx + \frac{1}{5}e^{2x}sinx$

Table (4 -2) compare the error by our method with Runge – kutta method in example (4-2)

xi	The Absolute Error (Runge – kutta)	The Absolute Error by our method
0	0	0
0.1	3.70e-03	6.011e-08
0.2	8.32e-03	8.090e-07
0.3	1.39e-02	5.161e-07
0.4	2.03e-02	6.059e-06
0.5	2.71e-02	6.058e-06
0.6	3.41e-02	7.591e-06
0.7	4.05e-01	6.052e-06
0.8	4.56e-01	6.301e-06
0.9	4.71e-01	6.054e-06
1	4.50e-01	6.057e-06

5. Mathematical model of Partial differential equations (PDEs)

The simple definition of partial differential equation (PDEs) is an equation which containing partial derivatives of one or more dependent variables with respect to independent variables more than one..In this section, we show the steps involved the mathematical model to solve partial differential equation (PDE) with initial condition. A partial differential equation (PDE) of order (n) for the function $u(x_1, x_2, \ldots, x_m)$ is an equation of the form

$$f(x_1, x_2, \ldots, x_m, \frac{\partial u}{\partial x_1}, \ldots, \frac{\partial u}{\partial x_m}, \frac{\partial^2 u}{\partial^2 x_1}, \frac{\partial^2 u}{\partial x_1 \partial x_2}, \ldots, \frac{\partial^2 u}{\partial^2 x_m}, \ldots, \frac{\partial^n u}{\partial^n x_1}, \ldots, \frac{\partial^n u}{\partial^{\alpha_1} x_{i_1} \partial^{\alpha_2} x_{i_2} \ldots \partial^{\alpha_k} x_{i_k}}) = 0 \qquad (5.1)$$

Where $\alpha_1 + \alpha_2 + \cdots + \alpha_k = n$, $1 \le i_j \le m$ *and* $0 \le k \le n$

With the initial value condition which has all of the conditions specified at the same value of the independent variable

$$f(x_1, x_2, \ldots, x_{k-1}, 0, x_{k+1}, x_{k+2}, \ldots, x_m) = 0 \qquad (5.2)$$

$$, k = 2,3, \ldots m-1 , \quad if\ m \ge 3$$

With $f(0, x_2, \ldots, x_m) = 0$ and $f(x_1, x_2, \ldots, x_{m-1}, 0) = 0$

Suppose the polynomial $P(x_1, x_2, \ldots, x_m)$ is a solution ,such

$$p(x_1, x_2, \ldots, x_m) = \sum_{k_1=0}^{m_1} . \sum_{k_2=0}^{m_2} . \ldots \sum_{k_m=0}^{m_m} d_{i_1 i_2 \ldots i_m} \psi_{k_1 k_2 \ldots k_m}(x_1, x_2, \ldots, x_m) \qquad (5.3)$$

The special case

$$\psi_{k_1 k_2 \ldots k_m}(x_1, x_2, \ldots, x_m) = x_1^{k_1} x_2^{k_2} \ldots x_m^{k_m} \qquad (5.4)$$

Then the partial derivate of the solution $P(x_1, x_2, \ldots, x_m)$ with (4.3) is

$$\frac{\partial^k p(x_1, x_2, \ldots, x_m)}{\partial^{\alpha_1} x_1 \partial^{\alpha_2} x_2 \ldots \partial^{\alpha_m} x_m} = \sum_{i_1=\alpha_1}^{m_1} \sum_{i_2=\alpha_2}^{m_2} . \ldots \sum_{i_m=\alpha_m}^{m_m} d_{k_1 k_2 \ldots k \alpha_1}(k_1!)(k_2!) \ldots (k_m!) \binom{m_1}{k_1} \binom{m_2}{k_2} \ldots \binom{m_m}{k_m} x_1^{k_1-\alpha_1} x_2^{k_2-\alpha_2} \ldots x_m^{k_m-\alpha_m} \qquad (5.5)$$

Where $\sum_{i=1}^{m} \alpha_i = k \qquad 0 \le \alpha_1 \le k \qquad (5.6)$

5-1: Remark: partial differential equation (PDE) of two variable and order (n)

in general form the linear PDE of two independent variables and order (n) is

$$f\left(x, y, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial^2 u}{\partial^2 y}, \frac{\partial^2 u}{\partial x \partial y}, , \frac{\partial^n u}{\partial^n y}, \right) = \sum_{k=1}^{6} a_k(x, y) \frac{\partial^h p(x, y)}{\partial^{\alpha_1} x \partial^{\alpha_2} y} - g(x, y) = 0 \qquad (5.7)$$

Where

$$h=\begin{cases}\left\lfloor\frac{1+\sqrt{1+8k}}{2}\right\rfloor-1 & if\ 1+8k\neq x^2 \quad \forall x\in Z\\ \left\lfloor\frac{1+\sqrt{1+8k}}{2}\right\rfloor-2 & if\ 1+8k=x^2 \quad \forall x\in Z\end{cases} \qquad (5.8)$$

$$\alpha_1+\alpha_2=h \quad ,and\ \ \alpha_1=k-\tfrac{1}{2}h(h+1)-1 \qquad (5.9)$$

In the case of the second order of the (PDE) the second order for the function u(x,y) is an equation of the form

$$f(x,y,\tfrac{\partial u}{\partial x},\tfrac{\partial u}{\partial y},\tfrac{\partial^2 u}{\partial^2 x},\tfrac{\partial^2 u}{\partial x\partial y},\tfrac{\partial^2 u}{\partial y\partial x},\tfrac{\partial^2 u}{\partial^2 y})=0 \qquad (5.10)$$

Suppose the polynomial p(x,y) is a solution of the PDE (5-1-8) of second order ,where

$$p(x,y)=\sum_{i=0}^{m_1}\sum_{j=0}^{m_2}d_{ij}\psi_{ij}(x,y) \qquad (5.11)$$

The special case $\psi_{ij}(x,y)=x^iy^j$ (5.12)

where d_{ij} are coefficients of the polynomial p(x,y) , and (m_1 +m_2) is the degree of the polynomial then

$$p(x,y)=\sum_{i=0}^{m_1}\sum_{j=0}^{m_2}d_{ij}x^iy^j \qquad (5.13)$$, then

$$\frac{\partial p(x,y)}{\partial x}=\sum_{i=1}^{m_1}\sum_{j=0}^{m_2}d_{ij}(i)x^{i-1}y^j \qquad (5.14)$$

$$\frac{\partial p(x,y)}{\partial y}=\sum_{i=0}^{m_1}\sum_{j=1}^{m_2}d_{ij}(j)x^iy^{j-1} \qquad (5.15)$$

$$\frac{\partial^2 p(x,y)}{\partial x\partial y}=\sum_{i=1}^{m_1}\sum_{j=1}^{m_2}d_{ij}(i)(j)x^{i-1}y^{j-1} \qquad (5.16)$$

$$\frac{\partial^2 p(x,y)}{\partial x^2}=\sum_{i=2}^{m_1}\sum_{j=0}^{m_2}d_{ij}(i)(i-1)x^{i-2}y^j \qquad (5.17)$$

$$\frac{\partial^2 p(x,y)}{\partial y^2}=\sum_{i=0}^{m_1}\sum_{j=2}^{m_2}d_{ij}(j)(j-1)x^iy^{j-2} \qquad (5.18)$$

5-2: The second order linear PDE

Consider a general second order linear equation in two independent variables

$$G(x,y)=a_1(x,y)u_{xx}+2a_2(x,y)u_{xy}+a_3(x,y)u_{yy}+a_4(x,y)u_x+a_5(x,y)u_y+a_6(x,y)u-g(x,y) \qquad (5.19)$$

Where u(x,y) is dependent variable on x and y ,the equation (5-1-17) can be written as

$$G(x,y)=\sum_{k=1}^{6}a_k(x,y)\frac{\partial^h p(x,y)}{\partial^{\alpha_1}x\partial^{\alpha_2}y}-g(x,y) \qquad (5.20)$$

Where $h,\alpha_1,\ and\ \alpha_2$, see the equation (5.8) and ((5.9) .can be written

$h=\lfloor\log_2 k\rfloor$,k=1,2,..,6 (5.21) and

$\alpha_1=k-2^h$ (5.22)

$\alpha_1+\ \alpha_1=h$ (5.23)

For example

if k=5,then $h=\lfloor\log_2 5\rfloor=2$ and $,\alpha_1=5-2^2=1$ $,\alpha_2=1$

and $\frac{\partial^h p(x,y)}{\partial^{\alpha_1}x\partial^{\alpha_2}y}=\frac{\partial^2 p(x,y)}{\partial x\partial y}$

To dividing points x_i and y_j by using the formula

x_i=ih ,y_j=jk i=1,2,…m_1 and j=1,2,…m_2 with

$h=\frac{1}{m_1}$ (5.24)

and $k=\frac{1}{m_2}$ (5.25)

then each coefficient of (5.13) is represented in binary numbers where fitness function fit(x,y) evaluation for approximation of coefficients corresponding to the chromosome

such $fit(x,y) = \min \sum_{i=1}^{m_1} \sum_{j=1}^{m_2} G(x_i, y_i)^2$ (5.26)

then

$$fitness = \frac{1}{1+fit(x,y)} \tag{5.27}$$

5.4:Example

Solve $\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = xe^y$

$$R = \left\{(x,y): \begin{matrix} 0 < x < 2 \\ 0 < y < 1 \end{matrix}\right.$$

With the boundary condition

$$u(0,y) = 0\,, u(2,y) = 2e^y \qquad 0 \le y \le 1$$

$$u(x,0) = x\,, u(x,1) = e^x \qquad 0 \le x \le 2$$

And the approximate the exact solution

$u(x,y) = xe^y \quad with\ h = 6\ and\ k = 5$ =5

Table (5-1) compare the error by our method with error by finite difference method

i	j	xi	yj	Error by finite difference method	Error by our method
1	1	0.3333	0.2000	1.30e-04	1.21e-06
1	2	0.3333	0.4000	2.82e-04	1.090e-06
1	3	0.3333	0.6000	1.6e-04	2.161e-06
1	4	0.3333	0.8000	2.5e-04	2.059e-06
2	1	0.6667	0.2000	4.01e-04	3.058e-06
2	2	0.6667	0.4000	4.31e-04	4.11e-06
2	3	0.6667	0.6000	3.15e-04	6.52e-06
2	4	0.6667	0.8000	3.46e-04	6.72e-06
3	1	1.0000	0.2000	5.71e-04	6.94e-06
3	2	1.0000	0.4000	6.24e-04	7.07e-06
3	3	1.0000	0.6000	4.51e-04	7.67e-06
3	4	1.0000	0.8000	4.27e-04	3.85e-06
4	1	1.3333	0.2000	6.79e-04	3.17e-05
4	2	1.3333	0.4000	7.53e-04	4.27e-05
4	3	1.3333	0.6000	5.41e-04	4.85e-05
4	4	1.3333	0.8000	3.71e-04	5.05e-06
5	1	1.6667	0.2000	5.85e-04	5.37e-05
5	2	1.6667	0.4000	6.16e-04	5.55e-05
5	3	1.6667	0.6000	4.81e-04	6.15e-05
5	4	1.6667	0.8000	4.89e-04	7.01e-05

6- Conclusion

In this study we introduced the method which propose a polynomial with using Genetic Algorithm (GA) to applied for the solving ordinary differential equations (ODE's) and partial differential equations (PDEs). It is observed that this method has general effect for applications. In this study we showed that the genetic algorithm can be used to find analytic solutions for differential equations and we can find the values of unknown function approximately. In this study we have been able to use the genetic algorithm instead of using the Evolution Strategies (ESs) to solve differential equations by proposing a polynomial as a solution to differential equations and by using the genetic algorithm; we can found the coefficients of this polynomial. This study can be expanded using grammatical evolution for the proposed solution of differential equation DEs. The study can also be expanded to be used to solve a system of ordinary differential equations

CONFLICT OF INTERESTS

There are no conflicts of interest.

7- Reference

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حل المعادلات التفاضلية باستخدام تحديث الخوارزمية الجينية

الخلاصة

المعادلة التفاضلية(DE) هي معادلة رياضياتية تحتوي على مشتقات كمتغير، ومن امثلتها المعادلات التي تمثل الكميات الفيزيائية، في هذه الورقة قدمنا تعديل على الطريقة التي تقترح بان يكون حل المعادلات التفاضلية الاعتيادية (EDO) من الدرجة الثانية على شكل متعددة الحدود وباستخدام الخوارزمية التطورية(ES)تجد معاملات الحل المقترح [١] .طريقتنا تقترح ايضا متعددة حدود لحل المعادلات التفاضلية الاعتيادية (EDO) ولكل الدرجات وليس للدرجة الثانية فقط ونستخدم الخورارزميه الجينية(GA) بدل الخوارزمية التطورية (ES) لايجاد معاملات متعددة الحدود ,وكذلك استخدمنا متعددة الحدود لحل المعادلات التفاضلية الجزئية(DEP) وباستخدام الخوارزمية الجينية (GA) لايجاد معاملات متعددة الحدود التي تمثل حل المعادلات التفاضلية الجزئية حيث تستخدم استراتيجيات التطورسلسلة من الخطوات التطورية المستندة دالة التقييم ومن خلال سلسلة من الطفرات على حل فردي وليس على مجموعة من الحلول على خلاف الخوارزمية الجينية [٢]. استخدمنا امثلة عددية تظهر دقة اسلوبنا مقارنة مع بعض الاساليب العددية المعروفة مع نسبة خطأ اقل بكثير مقارنة مع افضل الحلول بالطرق العددية

الكلمات الدالة: المعادلات التفاضلية الاعتيادية, الخوارزمية الجينية ,الحلول العددية ,المعادلات التفاضلية الجزئية.